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A Robust Control of Two-Wheeled Mobile Manipulator with Underactuated Joint by Nonlinear Backstepping Method

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1 Introduction

It is possible to increase the working area of the fixed based manipulator by combining them with the mobile platforms. Mobile manipulators have many degrees of freedom to perform manipulation and locomotion tasks such avoidance of obstacles, pushing, picking and placing task in daily life environments, and hazardous environments.

The mobile manipulators, which have three or more wheels, are statically stable. Therefore, they do not need any motion to maintain their balance during standing pose. However, they should have large bases and footprints to compensate the manipulator motion, keep from falling over and handle the heavy payloads. Two-wheeled systems are dynamically stable and need an active control mechanism to maintain their balance and pose. These systems have smaller footprint, and as a results, they are more flexible and maneuverable with respect to mobile manipulators.

This study is focused on one of these systems called two-wheeled mobile manipulator with an underactuated joint. This system is modeled as a virtual double inverted pendulum and used with the nonlinear backstepping based control design to stabilize the passive joint and control the posture of the manipulator.

2 Modeling

Model of the two-wheeled mobile manipulator is given in Figure 1. Since two-wheeled mobile manipulator is a complex structure, it is modeled as a virtual double inverted pendulum as shown in Figure 2. In this model, the second, third and fourth links of the manipulator are considered as the virtual second link of the double inverted pendulum model.

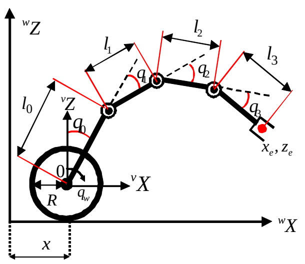


Figure 1: The Two-Wheeled Mobile Manipulator

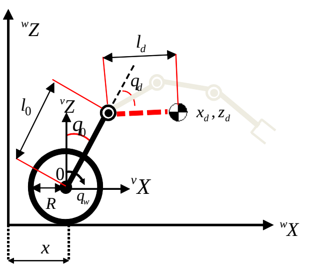


Figure 2: The Virtual Double Inverted Pendulum Model

3 Control System

3.1 *Control of the Virtual Double Inverted Pendulum*

Backstepping, which is a nonlinear recursive design methodology for tracking and regulation strategies, is utilized to stabilize the motion of the passive joint. To control the position of the passive joint, first error variable, its derivative and integral are defined for the first step of the backstepping method as

$$\begin{aligned} z_1 &= q_0 - q_0^{cmd} && (1)\\ \dot{z}_1 &= \dot{q}_0 - \dot{q}_0{}^{cmd} && (2)\\ \xi &= \int_0^t (q_0 - q_0^{cmd})d\tau && (3) \end{aligned}$$

Lyapunov function candidate for this error variable is selected as

$$V_1(\xi, z_1) = \frac{1}{2}(\lambda\xi^2 + z_1^2) \quad (4)$$

If $\dot{q}_0$ were selected as

$$\dot{q}_0 = -\lambda\xi - c_1 z_1 + \dot{q}_0{}^{cmd} \quad (5)$$

$\dot{V}_1$ becomes negative semi-definite and the error variable z_1 becomes stable. Indeed, $\dot{q}_0$ is not an actual control input and cannot be chosen as (5). Therefore, virtual control input α is selected as

$$\alpha = -\lambda\xi - c_1 z_1 + \dot{q}_0{}^{cmd} \quad (6)$$

Difference between the actual and virtual control input is defined as a second error variable z_2, which is given as

$$\begin{aligned} z_2 &= \dot{q}_0 - \alpha && (7)\\ &= \dot{q}_0 + \lambda\xi + c_1 z_1 - \dot{q}_0{}^{cmd} && (8) \end{aligned}$$

The new system can be written as below

$$\begin{aligned} \dot{\xi} &= z_1 && (9)\\ \dot{z}_1 &= -\lambda\xi - c_1 z_1 + z_2 && (10)\\ \dot{z}_2 &= \ddot{q}_0 - c_1^2 z_1 - c_1\lambda\xi + c_1 z_2 + \lambda z_1 - \ddot{q}_0^{cmd} && (11) \end{aligned}$$

In order to increase the robustness of the system against unknown disturbance, parameter variation, and modeling errors, sliding mode is utilized at the final step of the backstepping method. Sliding surface is defined as (12), which actually equals to the second error variable z_2.

$$\sigma = \dot{q}_0 - \alpha \quad (12)$$

New Lyapunov function is set as

$$V_2(\xi, z_1, \sigma) = V_1(\xi, z_1) + \frac{1}{2}\sigma^2 \quad (13)$$

Motion equation of the passive joint is obtained from dynamic equation of the virtual double inverted pendulum model as

$$\ddot{q}_0 = \gamma \ - \beta\ddot{q}_w - \eta \quad (14)$$

where

$$\begin{aligned} \gamma &= \frac{g\sin(q_0)}{l_0} && (15)\\ \beta &= \frac{R\cos(q_0)}{l_0} && (16)\\ \eta &= \frac{m_2 l_d(\cos(q_0 - q_d)\ddot{q}_d + \dot{q}_d\sin(q_0 - q_d))}{(m_0 + m_d)l_0} && (17) \end{aligned}$$

When acceleration input of the wheels $\ddot{q}_w$ are chosen as

$$\ddot{q}_w = \frac{1}{\beta}[\gamma + c_2\sigma + Ksgn(\sigma) + (1+\lambda)z_1 + c_1\dot{z}_1 - \ddot{q}_0^{cmd} - \eta] \quad (18)$$

then derivate of the V_2 becomes negative semi-definite as seen from (19), which indicates that V_2 is bounded with the error variables ξ, z_1, and σ.

$$\dot{V}_2(\xi, z_1, \sigma) = -c_1 z_1^2 - c_2\sigma^2 - K|\sigma| \leq 0 \quad (19)$$

By using Barbalat Lemma, it can be shown that error variables converge to zero as time goes to infinity.

In the wheel control, disturbance observer is used to realize the robust acceleration based backstepping control. The block diagram of the wheel control is given in Figure 3.

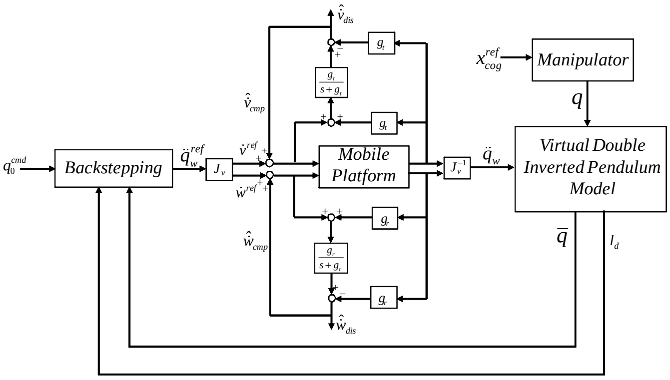


Figure 3: Block diagram of wheel control

3.2 Control of the Manipulator

The joint space acceleration reference of the redundant manipulator can be written as (20) by using workspace and joint space disturbance observer.

$$\ddot{\boldsymbol{q}}^{ref} = \boldsymbol{J}_w^+ \ddot{\boldsymbol{x}}_{cog}^{ref} + \ddot{\boldsymbol{q}}_{null}^{ref} \quad (20)$$

where $\boldsymbol{J}_w^+$ is the weighted pseudo inverse matrix and is defined as

$$\boldsymbol{J}_w^+ = \boldsymbol{W}^{-1}(\boldsymbol{J}_w^T \boldsymbol{J}_w \boldsymbol{W}^{-1} \boldsymbol{J}_w^T)^{-1} \quad (21)$$

$\boldsymbol{W}$ is a diagonal weighting matrix and selected to decrease the effect of the passive joint on motion of the manipulator. The acceleration references of the COG motion in the workspace $\ddot{\boldsymbol{x}}_{cog}^{ref}$ and in the null space $\ddot{\boldsymbol{q}}_{null}^{ref}$ are selected as

$$\begin{aligned} \ddot{\boldsymbol{x}}_{cog}^{ref} &= \ddot{\boldsymbol{x}}_{cog}^{cmd} + \boldsymbol{K}_p(\boldsymbol{x}_{cog}^{cmd} - \boldsymbol{x}_{cog}) \quad (22) \\ &\quad + \boldsymbol{K}_v(\dot{\boldsymbol{x}}_{cog}^{cmd} - \dot{\boldsymbol{x}}_{cog}) \\ \ddot{\boldsymbol{q}}_{null}^{ref} &= -\boldsymbol{K_{np}}\frac{\partial V(\boldsymbol{q})}{\partial \boldsymbol{q}} - \boldsymbol{K_n}\dot{\boldsymbol{q}} \quad (23) \end{aligned}$$

where $\boldsymbol{K}_p$ and $\boldsymbol{K}_v$ are the proportional and differential gains of the manipulator in the workspace. In (23), first term is the null space vector for the avoidance of singularity and second term is the velocity damping vector, used to increase the stability of null space motion. The block diagram of the manipulator control is given in Figure 4.

4 Experiment

Several experiments were performed to show the validity of the proposed method. In the first experiment, joint angle command of the passive joint is given as zero degree and sine wave trajectory is selected for the the COG command of the manipulator. Figure 5(a) shows the position response of the

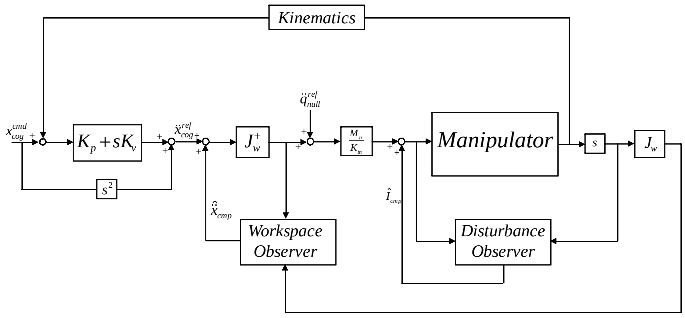
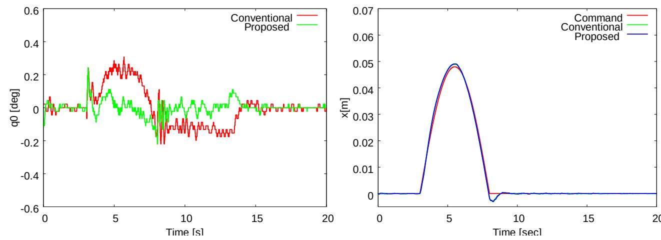


Figure 4: Block diagram of manipulator control system

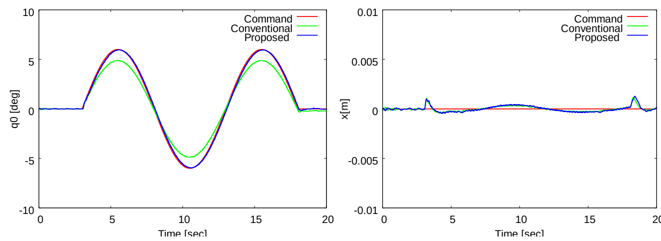
passive joint, and Figure 5(b) shows the position response of the COG of the manipulator in x direction, respectively.

In the second experiment, sine wave trajectory is selected for the joint angle command of the passive joint and the COG command of the manipulator. Joint angle response of the passive joint, and position response of the COG of the manipulator in x direction can be seen in Figure 6(a) and (b), respectively.



(a) Passive Joint (b) COG in x direction

Figure 5: The First Experiment Results



(a) Passive Joint (b) COG in x direction

Figure 6: The Second Experiment Results

5 Conclusions

Ttwo-wheeled mobile manipulator with an underactuated joint systen is modeled as the virtual double inverted pendulum and nonlinear backstepping is used to achive the stablity and position control of the passive joint. In the manipulator control, workspace is used to the COG of the manipulator and null space is utilized to avoid the singularities. Finally, the validity of the proposed method was shown by the experimental results.

References

[1] M. Krstic, I. Kanellakopoulos, and P. V Kokotovic *Nonlinear and adaptive control design,* Wiley, New York, 1995.

[2] T. Murakami, K. Kahlen, and R. W. D. Doncker "Robust Motion Control based Projection Plane in Redundant Manipulator", *IEEE Transaction on Industrial Electronics,* vol.49, no.1, pp. 248-255, 2002.